Using Intuitionistic Logic as a basis for Legal Ontologies

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Considerations on Legal Ontologies

What is an Ontology?

- A declarative description of a domain.
- Concretely, an Ontology is a Knowledge Base: A set of Logical Assertions that aims to describe a Domain completely.
- Consistency is mandatory.
- Consistency means absence of contradictions.
- Negation has an essential role.
Considerations on Legal Ontologies

What does it mean the term “Law”?

- What does count as the “unit of law”? Open question, a.k.a. “The individuation problem”.
- (Raz1972) What is to count as one “complete law”? 
Considerations on Legal Ontologies

Under Legal Positivism: Two main approaches to the “Individuation problem”.

1. Taking the collection of laws as a whole. A law, or general law, is a kind of deontic statement or proposition.

2. Taking into account all individually legal valid statement (ivls or vls for short) as individual laws. An individual law is not a deontic statement, it is not even a proposition. “The law” is the collection of all individual laws.
Considerations on Legal Ontologies

Formalization of Legal Ontologies according the second approach

- The first-class citizens of any Legal Ontology are vls. Only vls inhabits legal world. Influence of Kelsen’s characterization of law.

- There can be concepts on vls and relationship between vls. For example: PILBR, CIVIL, FAMILY, etc, can be concepts. LexDomicilium can be a relationship, a.k.a. a legal connection.

- Facilitates the analysis of structural relationship between laws, viz. Primary and Secondary Rules.

- It induces the natural precedence between laws. P.ex: “Peter is liable” precedes “Peter has a renting contract”.
Intuitionistic versus Classical logic:
What version is more adequate to Law Form. according our approach??

Classical Negation classifies: $\neg \phi \lor \phi$ is valid for any $\phi$

In $BR$, 18 is the legal age $BR$ contains all vls in Brazil

"Peter is 17"

“Peter is liable” $\notin BR$ iff “Peter is liable” $\in \neg BR$

Classical negation forces the existence of a liable Peter in some legal system abroad Brazil
Intuitionistic versus Classical logic:
What version is more adequate to Law Form. according our approach??

The Intuitionistic Negation

\[ \models_i \neg A, \text{iff, for all } j, \text{ if } i \preceq j \text{ then } \not \models_j A \]

\[ \not \models_i \neg \neg A \rightarrow A \text{ and } \not \models_i A \lor \neg A \]
Intuitionistic versus Classical logic:

What version is more adequate to Law Form. according our approach??

An Intuitionistically based approach to Law

“Peter is liable” $\not\in BR$

“Peter is liable” $\in \neg BR$ means

There is no vls in $BR$ dominating “Peter is liable”

neither “Peter is liable” $\not\in BR$ nor “Peter is liable” $\in \neg BR$
The logical framework for ontologies formalization

*iALC* and *ALC* have the same logical language

- Binary (Roles) and unary (Concepts) predicate symbols, $R(x, y)$ and $C(y)$.
- Prenex Guarded formulas ($\forall y (R(x, y) \rightarrow C(y))$, $\exists y (R(x, y) \land C(y))$).
- Essentially propositional (Tboxes), but may involve reasoning on individuals (Aboxes), expressed as “$i : C$”.
- Semantics: Provided by a structure $\mathcal{I} = (\Delta^\mathcal{I}, \preceq^\mathcal{I}, \cdot^\mathcal{I})$ closed under refinement, i.e., $y \in A^\mathcal{I}$ and $x \preceq^\mathcal{I} y$ implies $x \in A^\mathcal{I}$. “$\neg$” and “$\sqsubseteq$” must be intuitionistically interpreted.
- It is not First-order Intuitionistic Logic. It is a genuine Hybrid logic.
Deductive Reasoning in $iALC$

Usual Structural-Rules for Intuitionistic Logic

- **$\sqsubseteq$-I**
  \[
  \frac{\Gamma, x : C \Rightarrow x : C, \Delta}{\Gamma, \Gamma_1, \Gamma_2 \sqsubseteq D \Rightarrow \Delta}
  \]

- **$\sqsubseteq$-r**
  \[
  \frac{\Gamma_1 \Rightarrow C, \Gamma_2, D \Rightarrow \Delta}{\Gamma_1, \Gamma_2, C \sqsubseteq D \Rightarrow \Delta}
  \]

- **$\sqcap$-I**
  \[
  \frac{\Gamma, x : C \Rightarrow \Delta, \Gamma, x : D \Rightarrow \Delta}{\Gamma, x : (C \sqcap D) \Rightarrow \Delta}
  \]

- **$\sqcup$-I**
  \[
  \frac{\Gamma, x : C \Rightarrow \Delta, \Gamma, x : D \Rightarrow \Delta}{\Gamma, x : (C \sqcup D) \Rightarrow \Delta}
  \]

- **$\forall$-I**
  \[
  \frac{\Gamma, x : \forall R. C, y : C, xRy \Rightarrow \Delta}{\Gamma, x : \forall R. C, xRy \Rightarrow \Delta}
  \]

- **$\forall$-r**
  \[
  \frac{\Gamma, xRy \Rightarrow y : C, \Delta}{\Gamma \Rightarrow x : \forall R. C, \Delta}
  \]

- **$\exists$-I**
  \[
  \frac{\Gamma, xRy \Rightarrow \Delta}{\Gamma, x : \exists R. C \Rightarrow \Delta}
  \]

- **$\exists$-r**
  \[
  \frac{\Gamma \Rightarrow \Delta, xRy}{\Gamma \Rightarrow \Delta, x : \exists R. C}
  \]

- **$\in$-r**
  \[
  \frac{\Delta \Rightarrow x : A, A \Rightarrow B}{\Delta \Rightarrow x : B}
  \]
Using *iALC* to formalize Conflict of Laws in Space

### A Case Study

Peter and Maria signed a renting contract. The subject of the contract is an apartment in Rio de Janeiro. The contract states that any dispute will go to court in Rio de Janeiro. Peter is 17 and Maria is 20. Peter lives in Edinburgh and Maria lives in Rio.

Only legally capable individuals have civil obligations:

- **PeterLiable ⪯ ContractHolds@RioCourt**, shortly, *pl ⪯ cmp*
- **MariaLiable ⪯ ContractHolds@RioCourt**, shortly, *ml ⪯ cmp*

**Concepts, nominals and their relationships**

- **BR** is the collection of Brazilian Valid Legal Statements
- **SC** is the collection of Scottish Valid Legal Statements
- **PIL_{BR}** is the collection of Private International Law in Brazil
- **ABROAD** is the collection of VLS abroad Brazil
- **LexDomicilium** is a legal connection:

**Legal Connections** The pair *⟨pl, pl⟩* is in **LexDomicilium**
Non-Logical Axiom Sequents

The sets $\Delta$, of concepts, and $\Omega$, of $iALC$ sequents representing the knowledge about the case

$$\Delta = \begin{align*} ml : BR & \quad pl : SC & \quad pl \preceq cmp \\ ml \preceq cmp & \quad pl \text{ LexDom } pl \end{align*}$$

$$\Omega = \begin{align*} PIL_{BR} \implies BR \\ SC \implies \text{ABROAD} \\ \exists \text{LexD}_1. L_1 \ldots \cup \exists \text{LexDom}. \text{ABROAD} \cup \ldots \exists \text{LexD}_k. L_k \implies PIL_{BR} \end{align*}$$
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A Case Analysis

In Sequent Calculus

\[
\begin{align*}
&\frac{\Omega}{\Delta \Rightarrow pl : SC} && \frac{pl : SC \Rightarrow pl : A}{cut} && \Delta \Rightarrow pl : A \\
&\Delta \Rightarrow pl : A && \frac{\exists \text{LexD.A} \Rightarrow \exists \text{LexD.A}}{\exists - R} && \exists \text{LexD.A} \Rightarrow PL_{BR} \\
&\Delta \Rightarrow pl : \exists \text{LexD.A} && \exists \text{LexD.A} \Rightarrow BR \quad \frac{\Omega}{BMI_{BR} \Rightarrow BR} && \exists \text{LexD.A} \Rightarrow BR \\
&\Delta \Rightarrow pl : BR \\
\end{align*}
\]
Summary of the Approach

- Individual Legal Valid Statements are the individuals of the universe.
- **Concepts** are Classes of individual laws.
- **Roles** (relationships) between individuals laws denote kinds of **Legal Connections**
- Subsumptions and Negations are intuitionistically interpreted (**iALC**)
- Does it avoid the contrary-to-duty paradoxes of the Deontic approach?
Conclusions

- Seems to be adequate, at least according one philosophical and jurisprudence theory.
- Juridic cases can be analyzed in the ABOX.
- TBOX describes “The Law”.
- \( \preceq \) is not always specified at the level of the TBOX.
- It seems to scale, but there is no empirical evidence.
- (?) Is the coherence analysis easier?
- (?) Work out with “Hard juridical Cases”.
- (?) Can it be the kernel of a tool for helping Judge’s decision (not a Sentence maker!!!)
THANK YOU